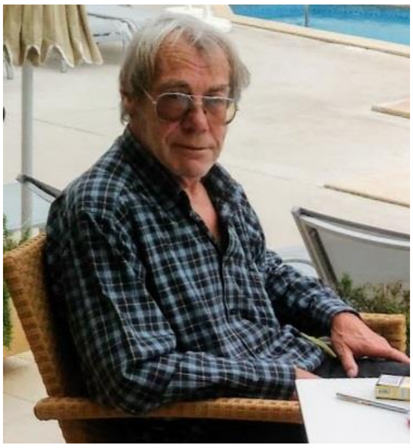

Professor Dr. Hanno Lefmann

Hanno Lefmann studied mathematics and physics at the University of Bielefeld, Germany, where he got his diploma in mathematics in 1982. He continued to be with the University of Bielefeld until 1992 and got there in mathematics his PhD in 1985 as well as his habilitation in 1992.

In 1992 he moved to the University of Dortmund at the Computer Science department, Germany, where he has been until 2000. He had visiting positions at IBM Scientic Center in Heidelberg, Germany, in 1989, and, within in the United States, at Georgia Institute of Technology as well as at Emory University both in Atlanta in 1989/1990 and at the University of Idaho in Moscow in 1992, and in 1999 at Chemnitz University of Technology, Germany.

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Topic: Testing and Estimating Graph Parameters for Monotone Properties

TESTING AND ESTIMATING GRAPH PARAMETERS FOR MONOTONE PROPERTIES

HANNO LEFMANN

Abstract

In the last years, there has been considerable interest in finding constant-time randomized sampling algorithms that estimate whether a given structure satisfies some property or does not, or to approximate the value of some numerical function associated with this structure. We focus in this talk on the graph case and consider algorithms that have the ability to query whether any chosen pair of vertices in the input graph is adjacent or not. We consider subsets $\mathcal{P}$ of the set $\mathcal{G}$ of all finite graphs, which are are closed under isomorphism; those are called *graph properties*. This includes all nontrivial *monotone* and *hereditary* graph properties, which are inherited by subgraphs and by induced subgraphs, respectively. A typical example of a monotone property is $\mathcal{P} = \mathit{Forb}(\mathcal{F})$ for a graph family $\mathcal{F}$, i.e., the class of all graphs that do not contain a graph $F \in \mathcal{F}$ as a subgraph. Elements of $\mathit{Forb}(\mathcal{F})$ are said to be *$\mathcal{F}$-free*.

A graph property $\mathcal{P}$ is said to be *testable* if, for every $\varepsilon > 0$, there exist $q_{\mathcal{P}} = q_{\mathcal{P}}(\varepsilon)$, the *query complexity*, and a randomized algorithm $\mathcal{T}_{\mathcal{P}}$, the *tester*, which performs at most $q_{\mathcal{P}}$ queries in the input graph, satisfying the following property. For an n-vertex input graph Γ, the algorithm $\mathcal{T}_{\mathcal{P}}$ distinguishes with probability at least $2/3$ between the cases in which Γ satisfies $\mathcal{P}$ (i.e., $\mathit{dist}(\Gamma, \mathit{Forb}(\mathcal{F})) \leq \varepsilon$) and in which Γ is ε-far (i.e., $\mathit{dist}(\Gamma, \mathit{Forb}(\mathcal{F})) > \varepsilon$) from satisfying $\mathcal{P}$, that is, no graph obtained from Γ by the addition or removal of at most $\varepsilon n^2/2$ edges satisfies $\mathcal{P}$.

For any monotone graph property $\mathcal{P}$, Alon, Shapira and Sudakov gave a natural algorithm that computes the distance from the induced sampled graph to $\mathcal{P}$. However, one disadvantage of this approach is that its analysis relies heavily on stronger versions of the Szemerédi Regularity Lemma. Therefore, their algorithm has a query complexity of order at least $\mathtt{TOWER}(\mathtt{poly}(1/\varepsilon))$, which is a tower of twos of height polynomial in $1/\varepsilon$.

Similarly, a function $z\colon \mathcal{G} \to \mathbb{Q}$ from the set $\mathcal{G}$ of graphs into the set of rational numbers is called a *graph parameter* if it is invariant under permutations of vertices. A graph parameter $z\colon \mathcal{G} \to \mathbb{Q}$ is *estimable* if for every $\varepsilon > 0$ and every large graph Γ, the value of $z(\Gamma)$ can be approximated up to an additive error of ε by an algorithm that only has access to a subgraph of Γ induced by a uniformly at random chosen set of $q_z = q_z(\varepsilon)$ vertices, the *sample complexity*.

We survey some recent results in this area. Amongst others, we introduce the notion of *f-recoverable* graph properties $\mathcal{P}$ for some function $f\colon (0,1] \to \mathbb{Q}$. For such a graph property $\mathcal{P}$ the graph parameter $\mathit{dist}(\Gamma, \mathcal{P})$ can be estimated within an additive error of ε with sample complexity $2^{\mathtt{poly}(f(\varepsilon/6)/\varepsilon)}$. We obtain similar results for the graph parameter, which, for a graph family $\mathcal{F}$, counts the number of $\mathcal{F}$-free subgraphs of an input graph Γ.

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